Magnetostatics

Def:- Steady currents produce magnetic fields that are constant in time, the theory of steady currents is called Magnetostatics.

Magnetic induction $\vec{B}$: If a moving charge, passing threw a point, experiences a sideways deflecting force then there is a magnetic field at that point.

This magnetic field is defined by a vector quantity $\vec{B}$, which is known as "Magnetic Induction".

If, In a magnetic field a test charge q_0 at a point P, experiences (with velocity $\vec{v}$) a deflection force $\vec{F}$, then magnetic Induction at that point is given by this eqn:

$$\boxed{\vec{F} = q_0 \vec{v} \times \vec{B}}$$

at Point P,

magnitude of magnetic induction

$$F = q_0 v B \sin\theta$$

$$B = \frac{F}{q_0 v \sin\theta}$$

Unit of magnetic induction:

In SI Unit - 'Weber / meter^2' or 'Tesla'.

In C.G.S Unit - Unit of B is 'Gauss'

1 Tesla = 10^4 Gauss

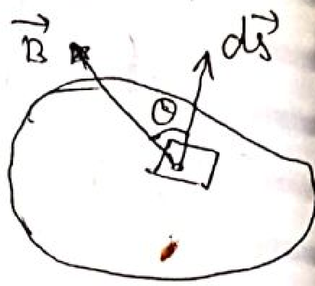
Lorentz Force:- If a charge q_0 moving in such a field, in which both magnetic field $\vec{B}$ and electric field $\vec{E}$ are exist, then at charge q_0, an force $\vec{F}$ is produces:-

$$\boxed{\vec{F} = q_0(\vec{E} + \vec{v} \times \vec{B})}$$

$\vec{v}$ is velocity of charge. $\vec{F}$ is said to be 'Lorentz force'.

Magnetic flux:- the total number of lines of magnetic induction passing threw a surface is said to be 'magnetic flux' passing threw that surface. it is denoted by Φ_B.

magnetic flux ϕ_B, passing threw total surface of area element $d\vec{s}$, is given by,



$$\boxed{\phi_B = \oint \vec{B} \cdot d\vec{s}}$$

Magnetic Induction $\vec{B}$ due to current distribution:

Bio Savert law:- In 1819 oersted ~~O~~bserved that wires carrying electric currents produced deflections of permanent magnetic Diboles placed in their neighborhood. Thus, the current were sources of magnetic flex density. Biot and Savart (1820), and Ampere (1820-25), established the basic experimental laws relating the magnetic induction $\vec{B}$ to the currents and established the law of force b/w one current and another. Steady currents produces magnetic fields that are constant in time;

Magnetostatics

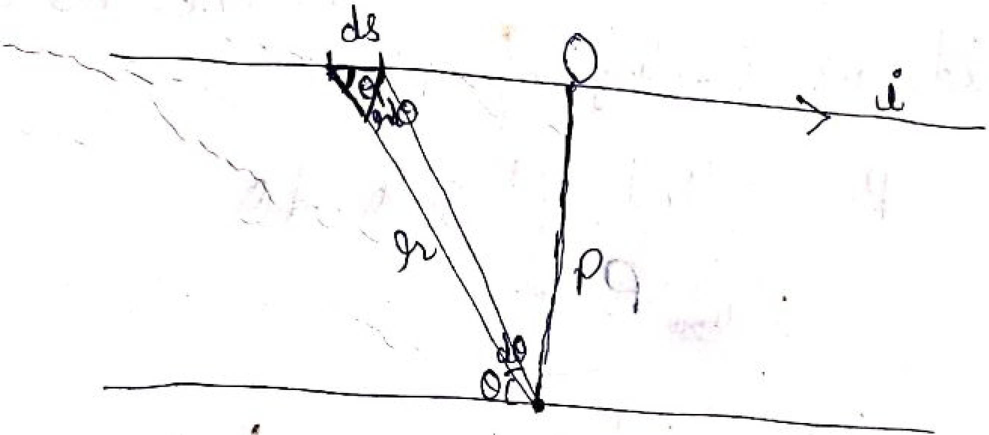
Steady state magnetic phenomena are characterized by no change in net charge density in space $\boxed{\nabla \cdot J = 0}$
Now, we will discuss connection b/w current & m. flux density ('B')

Magnectic Induction 'B' due to A current distribution :—

In early, 19th Century Two french Scientists, found a relationship b/w the current flowing along a wire and the magnetic field it produces.
The field due to a current i, flowing along an element of wire ds is given by

$$d\vec{B} = \frac{\mu_0}{4\pi r^3} i \,(d\vec{s} \times \vec{r}) \quad \text{—— (1)}$$

This is known as 'Bio-Savert law!
(m. f. be is at right angle to both $d\vec{s}$ and $\vec{r}$)



then

$$d\vec{s} \times \vec{r} = r \sin\theta \, ds$$

put in (1), then we have

$$d\vec{B} = \frac{\mu_0 i}{4\pi r^2} \sin\theta \, ds \quad \text{—— (2)}$$

from fig

$$\frac{P}{r} = \sin\theta \quad \text{———— (3)(a)}$$

$\because P$ is the perpendicular distance from the point where we are measuring the field due to the wire.

Similar,

$$\frac{r\,d\theta}{ds} = \sin\theta \quad \text{———— (3)(b)}$$

using 3(a) & 3(b), we may write

$$ds = \frac{P\,d\theta}{\sin^2\theta} \quad \text{———— (4)}$$

then by (2)

$$d\vec{B} = \frac{\mu_0 i}{4\pi r^2} \cdot \cancel{\sin\theta} \cdot \frac{P\,d\theta}{\sin^2\theta}$$

$$d\vec{B} = \frac{\mu_0 i \sin^{\cancel{2}}\theta}{4\pi \cancel{P^2}} \cdot \frac{P\,d\theta}{\cancel{\sin\theta}} \quad \text{———— (5)}$$

for a very long wire the value of θ for one end will be zero(0) while for other end will be π, the total field is (Integ. of dB).

$$B = \frac{\mu_0 i}{4\pi P} \int_0^{\pi} \sin\theta \cdot d\theta$$

$$\Rightarrow \frac{\mu_0 i}{4P}$$

$$\boxed{\vec{B} = \frac{\mu_0 i \cdot \cancel{2}}{\cancel{4}_2 \pi P}} \quad \text{———— (6)}$$

$$\boxed{\vec{B} = \frac{\mu_0 \cdot i}{2\pi P}} \quad \text{———— (6)}$$

If we draw an ~~imaginary~~ a closed surface around the wire, Q- what is net flux ~~is zero~~ (zero)

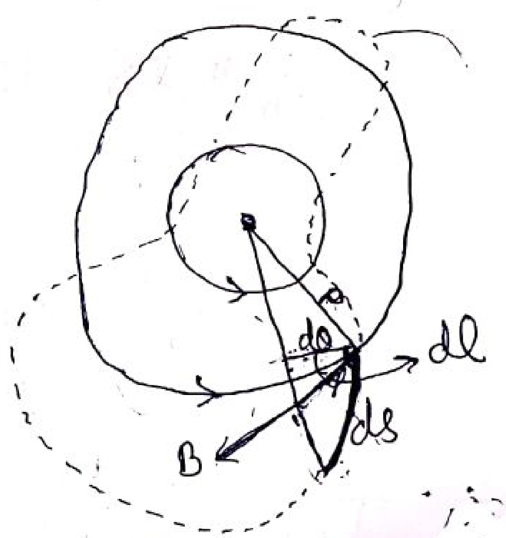
Since every exist point of a line has a Corresponding entry point of the lines.

i.e. $\nabla \cdot \underline{B} = 0$ (magnetic monopole do not exist)

(* div. means net flux outwards from the wire.)

Let us Consider,

A closed path which is around the wire, from this fig.,



$$\vec{B} \cdot d\vec{s} = B\, ds \cos\phi$$

$$= B\, dl$$

$$= B \rho\, d\theta$$

Surface Integral,

$$\int_S \vec{B} \cdot d\vec{s} = \int_0^{2\pi} \frac{\mu_0 i}{2\pi \rho} \cdot \rho\, d\theta$$

$$= \frac{\mu_0 i}{2\pi} \int_0^{2\pi} d\theta$$

$$= \frac{\mu_0 i}{2\pi} \cdot 2\pi$$

$$\int_S \vec{B} \cdot d\vec{s} = \mu_0 i \quad \text{——— (7)}$$

In terms of Current density (j)

$$\text{current } i = \int_A \vec{j} \cdot d\vec{A} \quad \text{——— (8)}$$

then,

$$\int_S \vec{B} \cdot d\vec{s} = \mu_0 \int_A \vec{j} \cdot d\vec{A} \quad \text{——— (9)}$$

using <u>Stokes's teorem</u>, we may write,

$$\int_S \vec{B} \cdot d\vec{s} = \int_A \text{curl}\, \vec{B} \cdot d\vec{A} \quad \text{——— (10)}$$

By (9) & (10)

$$\int_A \text{curl}\, \vec{B} \cdot d\vec{A} = \mu_0 \int_A \vec{j} \cdot d\vec{A}$$

diff. above eqn, we get

or, $\text{Curl}\, \vec{B} = \mu_0 \vec{j}$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$

This is known as Ampere's law in terms of current density.